

# INTERNATIONAL CONFERENCE ON ALGEBRA AND ITS APPLICATIONS

Southern Taiwan University of Technology, Tainan, Taiwan

December 19–24, 2005

## Abstracts

- Matej Brešar (Slovenia)

**Title:** *Commuting maps*

**Abstract.** A map  $f$  from a ring  $R$  into itself is said to be commuting if  $f(x)$  commutes with  $x$  for every  $x$  in  $R$ . We shall survey the development of the theory of commuting maps and their applications.

- Shun-Jen Cheng (Taiwan)

**Title:** *Canonical Basis in  $\Lambda^m(V) \otimes \Lambda^m(V)$*

**Abstract:** Let  $V$  is the natural  $U_q(\mathfrak{gl}(\infty))$ -module. In this talk we give combinatorial formulas for the canonical and dual canonical bases in the Fock space  $\Lambda^m(V) \otimes \Lambda^m(V)$ . These basis elements admit representation-theoretical interpretation as Kazhdan-Lusztig polynomials in a certain parabolic category of  $\mathfrak{gl}(m+n)$ -modules.

This is a joint work with W. Wang and R.B. Zhang.

- Alexandre Kirillov (USA)

**Title:** *Matrices with non-commuting entries and family algebras*

**Abstract.** Consider the matrix  $M = \|E_{ij}\|$  with entries in  $U(\mathfrak{gl}(n))$ . It satisfies the analogue of the Cayley-Hamilton identity:

$$M^N = \sum_{k=1}^N a_k M^{N-k}$$

where  $a_k$  are some elements of  $Z(\mathfrak{g})$ .

For a general matrix  $A$  with entries from a non-commutative algebra  $\mathcal{A}$  there is no reason to have something like this. There is however a case when the powers of  $A$  are linearly dependent

over the center  $Z$  of  $\mathcal{A}$ . It happens, when  $A$  is similar to a matrix with elements from  $Z$ . We show that it is exactly the case when we consider the matrices which belong to a so-called quantum family algebra:

$$Q_\lambda(\mathfrak{g}) = (U(\mathfrak{g}) \otimes \text{End}(V_\lambda))^G$$

Recall that a generic matrix over a commutative field can be reduced to the so-called **second normal form**:

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ a_n & a_{n-1} & a_{n-2} & \dots & a_1 \end{pmatrix}.$$

This property also generalizes to elements of a quantum family algebra.

Recently in the joint paper with L. Rybnikov we introduced the notion of odd family algebras.

I hope that the study of family algebras will help to understand the structure of  $U(\mathfrak{g})$  for semisimple Lie algebras  $\mathfrak{g}$  and some other non-commutative objects.

- Ching-Hung Lam

**Title:** *Ising vectors and McKay's  $E_8$  observation on the Monster simple group*

**Abstract.** In this talk, we shall discuss certain weight two vectors (called Ising vectors) in the VOA  $V_{\sqrt{2}} E_8$  and  $V_A^+$ . We shall discuss McKay's  $E_8$ -observation on the Monster simple group using these Ising vectors.

- Leonid G. Makar-Limanov (USA)

**Title:** *Ten years of AK-invariant: successes and disappointments*

**Abstract.** AK invariant was introduced about ten years ago in order to prove that certain three-folds are exotic structures: it means that they are very similar to the usual three-dimensional space and that difference is too subtle to notice it by looking at small neighborhoods of points.

The invariant turned out to be useful in other settings as well and allowed to obtain some new results and new proofs of old results which are by far more simple than the proofs known before.

But unfortunately we still cannot answer a lot of natural questions about the behavior of the invariant.

- Edmund R. Puczyłowski (Poland)

**Title:** *Around Koethe's nil ideal problem*

**Abstract.** In 1930 Köthe asked whether every left ideal of an arbitrary associative ring contained

*in the nil radical of the ring?* Though this question looks very elementary and was raised more than seventy years ago it is still open. It is called **Koethe's nil ideal problem** and is one of the most famous open problems in ring theory. Attempts to solve it led to many interesting, deep and sometimes surprising results. There are also many related open problem. The aim of the talk is to present several such problems as well as some new results obtained in the area. In particular it will be shown that Koethe's problem is equivalent to a problem raised in 1969 by Andrunakievich.

- Weiqiang Wang (USA)

**Title:** *The Bloch-Okounkov correlation functions and generalizations*

**Abstract:** Bloch and Okounkov introduced a certain  $n$ -point correlation functions on a Fock space several years ago and found a remarkable closed formula in terms of theta functions. These  $n$ -point functions admit geometric interpretations in terms of Gromov-Witten theory and Hilbert schemes of points. After reviewing some of these results, I will formulate several natural generalizations of these  $n$ -point functions (with motivations from twisted Fock spaces, vertex operators, and Macdonald polynomials, etc) and present closed formulas in some cases.

This is a joint work with Shun-Jen Cheng.

- Eugene Xia (Taiwan)

**Title:** *The mapping class group action on the moduli of representations*

**Abstract.** Let  $S$  be an oriented surface of genus  $g$  with boundary  $C$  and  $K$  a compact Lie group. Then  $K$  acts by conjugation on  $\text{Hom}_C(\pi_1(S), K)$ , the space of representations of the fundamental group  $\pi_1(S)$  into  $K$ . The resulting quotient is the moduli of representations  $\text{Hom}_C(\pi_1(S), K)/K$ . This moduli space has a canonical topology and measure. The mapping class group of  $S$  acts on  $\pi_1(S)$ , hence, acts on  $\text{Hom}_C(\pi_1(S), K)/K$  and one may study both the topological and measure dynamics of this action. This talk will focus on the measure dynamics of this action. We shall also outline a few results in topological dynamics if time permits.

- Jing Yu (Taiwan)

**Title:** *Algebraic relations among special values in characteristic  $p$*

**Abstract.** The goal of transcendence theory is to determine completely the algebraic relations among special values which naturally arise from arithmetico-geometric structures. In the positive characteristic world, the arithmetico-geometric structures are those from global function fields. Here more advances can be made. We will report on two cases where the algebraic relations are completely determined recently, one is certain zeta values, the other is on Gamma values. We will also report on the algebraic tools developed so far for the solution of our problems.

- Jiu-Kang Yu (USA)

**Title:** *Quasi-reductive group schemes*

**Abstract:** We study group schemes over a discrete valuation ring with reductive groups as generic fibers and reduced special fibers, and applications to the construction of Chevalley group by Mirkovic and Vilonen.

This is a joint work with G. Prasad.

- Ruibin Zhang (Australia)

**Title:** *A unified treatment of Schur-Weyl dualities for Lie algebras and for quantum groups*

**Abstract.** Let  $G$  be the complex Lie group  $GL_n$ ,  $O_n$ ,  $Sp_{2n}$ , or  $G_2$ , and let  $V$  be the natural  $G$ -module if  $G$  is a classical group, or the 7-dimensional irreducible module of  $G_2$ . We give a unified description of the endomorphism algebra  $\text{End}_G(V^{\otimes k})$  for any non-negative integer  $k$  by using the associative algebra  $T_k$  of “infinitesimal braids” due to Kohno and others. More specifically, there is a representation  $\Psi$  of  $T_k$  such that  $\text{End}_G(V^{\otimes k}) = \Psi(T_k)$ . The algebra  $T_k$  and the construction of the representation  $\Psi$  will be described, and a proof of the claimed equality will be given. The generalization of the result to quantum groups will also be presented.

This is joint work with G. I. Lehrer.