INTERNATIONAL CONFERENCE ON ALGEBRA
IN MEMORY OF KOSTIA BEIDAR
National Cheng Kung University, Tainan
March 6–12, 2005

DERIVATIONS AND SKEW POLYNOMIAL RINGS

CHEN–LIAN CHUANG
TEIPEI, TAIWAN
(WITH TSIU–KWEN LEE)

Let R be a prime ring and d a derivation of R. Let $\mathrm{E}(R,+)$ denote the ring of additive endomorphisms of R endowed with pointwise addition and composition multiplication. Obviously, $d \in \mathrm{E}(R,+)$. For $a \in R$, let $a_L \in \mathrm{E}(R,+)$ denote the left multiplication $a_L \colon x \in R \mapsto ax \in R$. Let S be the subring of $\mathrm{E}(R,+)$ generated by d and all a_L, $a \in R$. We shall compute the prime radical and minimal prime ideals of S as follows: Let $R[x;d]$ be the skew polynomial ring with the multiplication rule: $xr = rx + d(r)$ for $r \in R$. Since $da_L = d(a)_L + a_L d$ for $a \in R$, the map

$$\varphi : a_0 x^n + \cdots + a_{n-1}x + a_n \mapsto (a_0)_L d^n + \cdots + (a_{n-1})_L d + (a_n)_L \in S$$

is a surjective ring homomorphism. φ. Then $R[x;d]/\mathscr{A} \cong S$, where $\mathscr{A}$ is the kernel of φ. Let $\mathscr{P}$ be the ideal of $R[x;d]$ including $\mathscr{A}$ such that $\mathscr{P}/\mathscr{A}$ is the prime radical of $R[x;d]/\mathscr{A}$. Our aim is to describe in the ring $R[x;d]$ the ideals $\mathscr{A}$, $\mathscr{P}$ and also the minimal prime ideals over $\mathscr{A}$ in the following way: Let Q denote the symmetric Martindale quotient ring of R. The center C of Q is called the extended centroid of R. Let $C^{(d)} := \{\alpha \in C : d(\alpha) = 0\}$. Matczuk has shown that the center of $Q[x;d]$ assumes the form $C^{(d)}[\zeta]$. Given $f(\zeta) \in C[\zeta]$, we let

$$\langle f(\zeta)\rangle := R[x;d] \cap f(\zeta)Q[x;d].$$

We show that $\mathscr{A}$, $\mathscr{P}$ and all minimal prime ideals over $\mathscr{A}$ are of the above form and we compute these center elements $f(\zeta)$ explicitly.

Here is an application: Assume that d is a nilpotent derivation. Let m be the least integer such that $d^m(R)c = 0$ for somr $0 \neq c \in R$. Let (x^m) denote the ideal of $R[x;d]$ generated by x^m. Then $(x^m) \cap R = 0$. We extend (x^m) to an ideal $\mathscr{M}$ of $R[x;d]$ maximal with respect to the property $\mathscr{M} \cap R = 0$. The quotient ring $R[x;d]/\mathscr{M}$ is an extension of R and is called the d-extension of R. Using the above results, we show that $\mathscr{M} = \mathscr{P}$. So the d-extension of R exists uniquely and is isomorphic canonically to the quotient ring of S modulo its prime radical.