

KOSTIA'S CONTRIBUTION TO RADICAL THEORY AND RELATED TOPICS I, II

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One of Kostia's many research fields was the theory of radicals, concrete and general ones, and the structure of rings which are radical-free relative to a certain radical or which arise from certain radical rings. His contribution to this area and its impact to further researches were considerable, substantial and important; it is difficult to cover it even in two talks.

We intend to give an overview on a segment of his contribution to algebra, and shall address the following main points of his contribution:

1. GENERAL RADICAL THEORY

- The Suliński-Anderson-Divinsky Problem on the termination of Kurosh' lower radical construction at a given number of steps.
- Characterization of special radicals; constructing disjoint classes of rings which yield the same special radical as the upper radical of these classes.
- Constructing supernilpotent normal radicals which are non-special.

- Dependence and independence among radicals involving one-sided ideals (left and right hereditariness and subhereditariness, stability, strength and normality).
- Explicit description of radical classes with semisimple essential cover.
- Lattices of radicals (atoms, complements and the problem whether strong radicals form a sublattice of the lattice of all radicals)

2. CONCRETE RADICALS AND THE STRUCTURE OF RINGS

- On the Jacobson radical of finitely generated algebras.
- Koethe's nil ideal problem and the structure of some concrete radicals of polynomial rings.
- On the Behrens' radical of matrix rings.
- Kasch' total radical. Rings with zero total.

3. RINGS WITH INVOLUTION

- Rings with involution satisfying dcc on $*$ -biideals.
- Rings with involution satisfying acc on $*$ -biideal; an involutive counterpart of Hilbert's Basis Theorem.
- Rings with involution and primitivity.

4. NONASSOCIATIVE RINGS

- Sufficient conditions for a well-behaved radical theory.
- The splitting of the torsion radical in alternative (and Jordan) rings with dcc on principal right ideals.